LETTERS TO THE EDITOR

To the Editor:

In "Wavelet-Based Modulation in Control-Relevant Process Identification" (Feb. 1998), Carrier and Stephanopoulos compared three methods of obtaining third-order reduced models for the fifth-order linear time-invariant transfer function given by

$$g(z) = 10^{-5}\left[\frac{0.0052z^4 + 0.1316z^3 + 0.3307z^2 + 0.1302z + 0.005}{z^5 - 4.72z^4 + 9.14z^3 - 9.08z^2 + 4.63z - 0.969}\right]$$

where the sign of the last term in the denominator has been corrected. The five poles of the transfer function are: $0.87094 \pm 0.44323i$, $1.0374 \pm 0.21699i$, and 0.90323, and the four zeros are: −22.535, −2.2984, −0.4315, and −0.04302. The zeros are quite close to those given by Carrier and Stephanopoulos, but the poles differ somewhat from those given in the article.

The authors did not consider an easy way of obtaining a good reduced model by optimization in the frequency domain as proposed by Luus (1980). For the z-transfer function, we simply use the link between the z-variable and s-variable as shown by Yang and Luus (1983). By using this approach with the direct search procedure of Luus and Jaakola (1973) with 100 values chosen for the frequency between 0.01 and 3.20 through the increment factor of 1.06, the following reduced model was obtained

$$g_r(z) = 10^{-5}\left[\frac{-15.44026z^2 + 32.17736z - 13.77451}{z^3 - 2.97902z^2 + 2.99881z - 1.01488}\right]$$

The poles of $g_r(z)$ are 0.89936, $1.0398 \pm 0.21726i$, and the zeros are 0.60195, 0.14820.

This reduced model differs quite substantially from the one obtained by Carrier and Stephanopoulos (1998) by using wavelets

$$g_w(z) = 10^{-5}\left[\frac{2.43z^2 - 3.25z + 3.39}{z^3 - 2.98z^2 + 2.97z - 0.986}\right]$$

with poles at 0.85617, $1.0619 \pm 0.15485i$, and zeros at $0.66872 \pm 0.97359i$.

The real differences of these reduced models can be observed in the Nyquist plots, where in Figure 1, $g_r(z)$ is seen to represent the original system very well from $\omega = 0.01$ to $\omega = 0.350$. In Figure 2, where $g_w(z)$ is compared to the original transfer function, it is clear that the agreement is not as good. Even in the range of $\omega = 0.194$ to $\omega = 0.350$, where the arc seems to be approximated reasonably well, the frequencies are off.

The most surprising aspect of the article is that the reduced model obtained by least squares is so poor. The resulting model should be very close to $g_r(z)$. It is clear that there is something wrong with the establishment of the reduced model by Carrier and Stephanopoulos who reported

$$g_{LS}(z) = 10^{-5}\left[\frac{0.0733z^2 + 0.212z + 0.627}{z^3 - 2.95z^2 + 2.91z - 0.955}\right]$$

with poles at 0.83213, $1.0589 \pm 0.16221i$, and zeros at $-1.4461 \pm 2.5422i$. Perhaps, part of the difficulty arises from having taken such a huge number (4,096) of data points, when fewer than 100 would suffice. As was shown by Corlis and Luus (1971), even in presence of noise and modeling inaccuracies, a relatively short data length gives excellent results. In the article by Carrier and Stephanopoulos (1998) there are some obvious inaccuracies in the calculation of roots of polynomials, which could have been avoided by using a reliable root-solving routine, such as given by Luyben (1990). In their reduced models there is an inadequate number of figures given in the coefficients in the denominator terms. All one has to do is to put z to 1 to realize that there is no more than a single figure of accuracy in the denominator for obtaining the steady-state gain. Therefore, a better comparison should be used to evaluate the application of the wavelet approach for model reduction. In the meantime, optimization in the frequency domain continues to be a useful method for model reduction.

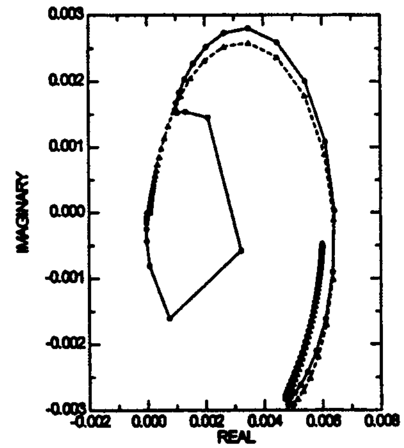

Figure 1. Nyquist plot of the original system and the reduced model $g_r(z)$ obtained by LJ optimization procedure.
●-●-● Original system: ▲--▲--▲ reduced model $g_r(z)$

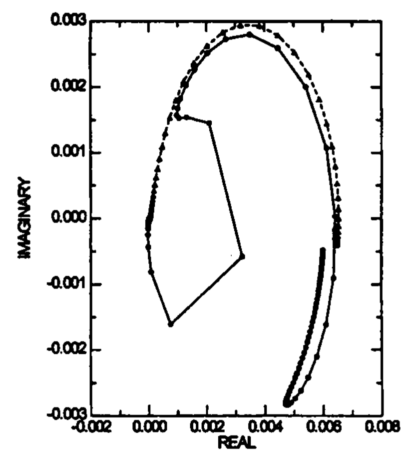

Figure 2. Nyquist plot of the original system and the reduced model $g_w(z)$ obtained by Carrier and Stephanopoulos (1998).
●-●-● Original system: ▲--▲--▲ reduced model $g_w(z)$

Literature cited

Carrier, J. F., and G. Stephanopoulos, "Wavelet-Based Modulation in Control-Relevant Process Identification," *AIChE J.*, **44**, 341 (1998).

Corlis, R., and R. Luus, "Compensation for Unknown Input Disturbances in Multiple-Input Identification and Optimal Control," *Can. J. Chem. Eng.*, **49**, 853 (1971).

Luus, R., "Optimization in Model Reduction," *Int. J. Control.*, **32**, 741 (1980).

Luus, R., and T. H. I. Jaakola, "Optimization by Direct Search and Systematic Reduction of the Size of Search Region," *AIChE J.*, **19**, 760 (1973).

Luyben, W. L., *Process Modeling, Simulation, and Control for Chemical Engineers*, McGraw-Hill, New York, p. 360 (1990).
Yang, S. M., and R. Luus, “A Note on Model Reduction of Digital Control Systems,” *Int. J. Control*, **37**, 437 (1983).

Rein Luus
Dept. of Chemical Engineering
University of Toronto
Toronto, ONM5S 3E5, Canada

Reply:

Carrier and Stephanopoulos (1998) *is not an article on how to obtain reduced-order models from higher-order ones*, as the letter by Luus might imply in the mind of a reader unfamiliar with the details of the article. It is an article on how to develop process models for the design of controllers, taking into account the localized character (localized in both time and frequency) of process input-output data, used in process identification.

Luus’ letter has focused on a very small part of the article by Carrier and Stephanopoulos (1998), specifically Example 1, where low-order models are generated from input-output data by various methods. Therefore, we will limit our response to the comments pertaining to this example.

In Example 1 the idea is as follows: We “know” that the process model is of high order, and we would like to create a low-order model for the design of a controller by emphasizing the accurate reproduction of process dynamics near the cross-over frequency. Although we use an explicit model to represent the underlying dynamics of the “unknown” process, nowhere in the numerical studies of Example 1 do we use the detailed model. In other words, the fifth-order linear model of the “actual” process is used only to generate output information for a given set of input values.

In view of this, Example 1 *does not try to advance a new method for model reduction*, but, instead, it studies the effectiveness of various filtering techniques in generating the input-output data for the identification of process models from the same original set of input-output data. This is the reason that Carrier and Stephanopoulos (1998) did not extend their discussion to the various model-reduction techniques, such as the work of Luus (1980). The use of the term “identification of reduced-order models” in the heading of Example 1 may have been an unfortunate one, but it was meant to convey the idea of generating lower-order models from the assumed order of the particular process. We note that througout the article, in all subsequent examples, the same principle has prevailed, that is, the models of the “unknown” processes have been used for simulation purposes only to provide the process output values only.

The above distinction is crucial. In his letter, Luus has recalled a method that he developed (Luus, 1980) for the generation of reduced-order models from higher-order ones, using optimization in the frequency domain. In this article, Luus has made full use of the knowledge of the higher-order model and not just of the model’s output values. In other words, Luus has used directly the structure of the process’ transfer function, that is, order and parametric values, in the ensuing optimization problem. This is far richer information than the one used by Carrier and Stephanopoulos (1998) for the Example 1, and, as information theory dictates, the model he has generated for Example 1 should be better.

Consequently, it is not a question of whether one uses optimization in the frequency domain or makes use of wavelet decomposition, as Luus implies in his concluding sentence, but a question of what problem one solves and how much information about the “unknown” process one uses. Carrier and Stephanopoulos (1998) “assumed” the unknown process to be of high order and examined the construction of fixed low-order models, without assuming knowledge of the process’ transfer function. We believe that this is closer to the realities of the process control practice.

In conclusion, we believe that Luus’ main argument is off the mark because he has wrongly interpreted what the objective of Example 1 was. He has compared apples and oranges.

Besides the main argument discussed above, in his letter Luus has made some additional comments, which require a response:

(1) He finds surprising the fact that the least-squares generated model (without any filtering of the output values) in Example 1 of Carrier and Stephanopoulos (1998) is so poor. It should not be actually surprising. Least-squares models are generated by minimizing the energy of the difference of the actual process output and of the model-generated output. If most of this signal’s energy is concentrated away from the cross-over frequency, and the degrees of freedom of the model are restricted (that is, the model-order is fixed), the behavior at the cross-over frequency will not have much impact on the least-squares metric, and, thus, it will lead to a model with a large relative error near the cross-over frequency. This is well known, has long been recognized, and has led to the use of band-pass filters, which improve the accuracy of the generated models near the cross-over frequency. This is exactly the context of Example 1, where the band-pass filter with least squares performs very well near the cross-over frequency.

(2) Luus has also indicated that the number of input and output values used by Carrier and Stephanopoulos (1998) in Example 1 (4,096) is very large and that fewer than 100 would suffice. He has not noticed that Carrier and Stephanopoulos have used the wavelet coefficients at the scale $j = 6$, where the actual number of points used for the estimation of the model parameters is 64.

(3) Luus is correct in that Carrier and Stephanopoulos used an inadequate number of significant figures in describing the values of the parameters in the generated model, and that the numerical values of the poles and zeros for the generated model could be estimated more accurately (which we have confirmed as a result of this letter). However, both of these are inconsequential to any aspects of the above discussion.

Literature cited

Carrier, J. F., and G. Stephanopoulos, “Wavelet-Based Modulation in Control-Relevant Process Identification,” *AIChEJ.*, **44**, 341 (1998).
Luus, R., “Optimization in Model Reduction,” *Int. J. Control.*, **32**, 741 (1980).

George Stephanopoulos and John Carrier
Dept. of Chemical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139